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A Multi-objective Multi-item Capacitated Lot-sizing Model with Safety Stock and Shortage Costs based on Just-in-time Approach

Afshar Bazyar[1], Ghasem Taghizade[2]
[1]Assistant Professor, Malek Ashtar University of Technology
[2]Associated Professor, Malek Ashtar University of Technology

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Corresponding Author:
Afshar Bazyar

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ABSTRACT

Lot-sizing problem is a class of production planning problems in which the availability amounts of the production plan are always considered as decision variable. Goal of this paper is to propose a new multi-item capacitated lot-sizing problem (MICLSP) with setup times, safety stock deficit costs, demand shortage costs both backorder and lost sale states, and different production manners. Although a considerable amount of researches concentrates on model development and solution procedures in the terms of single-objective problems in the past decade, to make the model more realistic, this paper develops a multi-objective mathematical programming model with three conflicting objectives. First objective attempts to minimize the total cost considered by the production plans including production costs with different production manners, inventory costs, safety stock deficit costs, shortage costs, and setup costs. Second objective is for leveling the production volume in different production periods. Third objective follow to force the model to produce as near as possible to just-in-time (JIT). The proposed model was indicated to be strongly NP-hard; hence a random search algorithm namely multi-objective simulated annealing (MOSA) has been proposed based on Lp-metric technique. At the end, results analysis on different sizes of problems demonstrates the intelligence and efficiency of the proposed methodology.

I. INTRODUCTION AND BACKGROUNDS

P Production planning problem consists in deciding how to transform raw material into final goods as to satisfy demand at minimum cost. The lot-sizing problem (LSP) is a crucial step and well-known optimization problem in production planning in which involved time-varying demand for set of *N* items over *T* periods. In industrial applications, several factors may sophisticate making best decisions. For instance, considering multi-items can be led to impossibility to satisfy demand. Moreover, safety stock is also a complicating constraint as a target to reach rather that an industrial constraint to satisfy (Tempelmeier and Derstroff, 1996).

Production planning typically includes three time scopes for decision making: long-term, medium-term and short-term. In long-term planning, the concentration mostly involves such strategic decisions as product, equipment, facility location, and resource planning. Medium-term planning often involves making decisions on material requirements planning, determining production quantities, and lot-sizing decisions during the planning period. In short-term planning, decisions usually involve daily scheduling of operations such as job sequencing or control in a workshop (Karimi et al., 2003). This paper concentrates on medium-term production planning and lot-sizing decisions.

In the former, Wagner and Within (1958) and Manne (1958) introduced various type of lot-sizing problems in terms of model development and solving methodologies. Following these, the single-item problem has been provided special interest for its relative simplicity and for its importance as a sub-problem of some more complicated lot-sizing problems (Kazan et al., 2000). In the literature, production planning models involve multiple items, restrictive capacities, and significant setup times which occurred frequently in industrial situations to determine optimal outputs. Loparic et al. (2001) developed valid inequalities for the single-item un-capacitated lot-sizing problem with sales instead of fixed demands and lower bounds on stock variables.

Aksen et al. (2003) introduced a profit maximization version of the well-known Wagner-Whitin model for the deterministic uncapacitated single-item lot-sizing problem with lost sales. Absi and Kedad-Sidhoum (2007) proposed a MCLSP with setup times and safety stock in which demand can be totally or partially lost. They also presented mixed integer programming heuristics based on a planning horizon decomposition strategy to find a feasible solution. Following this, Absi and Kedad-Sidhoum (2008) developed above model with considering shortage costs. Moreover, they presented fast combinatorial separation algorithm within branch-and-cut framework to solve the proposed model. Absi and Kedad-Sidhoum (2009) proposed MCLSP with setup times, safety stock deficit costs, and demand shortage costs. To solve their model, they first proposed a Lagrangian relaxation of the resource capacity constraints; then, a dynamic programming algorithm is developed to solve the induced sub-problem. Absi et al. (2013) proposed the multi-item capacitated lot-sizing problem with setup times and lost sales. To find feasible solutions, they proposed a non-myopic heuristic based on a probing strategy and a refining procedure. They also propose a meta-heuristic based on the adaptive large neighborhood search principle to improve solutions.

Regarding to expanding applicability of these problems in industrial operations, LSPs represent challenges to solve owing to its combinatorial nature. Chen and Thizy (1990) proved that the MICLSP with setup times is strongly NP-hard. Many researchers have attempted to solve MICLSP to very close to optimality (Tempelmeier and Derstroff, 1996; Sural et al., 2009). Thus, they were not quite successful in solving large-scale problems because of they could not anticipate the number of cutting planes that need to be generated, or the number of iterations that are required in a branch and bound approach. One group of researchers develops heuristics to solve large-scale problems (Tang, 2004; Berretta and Rodrigues, 2004; Han et al., 2009; Xiao et al., 2011). Nowadays, many realistic problems are involved simultaneous optimization of several objectives (Coello et al., 2007). Regarding the aforementioned multiple-objective LSPs works, a vast variety of solution methodologies including exact and approximation techniques has been utilized to find Pareto solution sets of different multi-criteria lot-sizing models.

In this paper, we follow to propose a new MICLSP with setup times, safety stock deficit costs, demand shortage costs both backorder and lost sale states, and different production manners. As main contribution in the model formulation area, this paper develops a multi-objective mathematical programming model with three conflicting objectives to make the model closer to reality. In the objectives, we include (I) minimizing the total cost considered by the production plans including production costs with different production manners, inventory costs, safety stock deficit costs, shortage costs, and setup costs; (II) minimizing required storage space. As regards proposed model is NP-hard, we propose multi-objective simulated annealing (MOSA) algorithm based on Lp-metric technique. Thus, these gaps cause to make a research question for proposing this model and solving it efficiently.

Rest of the paper is organized as follows: Section II provides the proposed problem definition and mixed-integer-programming formulation. In Section III, an multi-objective meta-heuristic algorithms are illustrated in details. Section IV provides the results of all solving methodologies statistically and graphically. Finally, Section V gives the conclusion and implications for future works.

II. PROBLEM FORMULATION

Many real-world problems involve simultaneous optimization of several objectives. In this type of optimization problems, there is usually no single optimal solution. Hence, all objectives are considered when a set of alternative solutions are optimal in the wider sense which no other solutions in the search space are superior to them. They are known as Pareto-optimal solutions (Coello et al., 2007). Therefore, a general multi-objective problem could be defined which is minimize a function *f(x)*, with P ($P > 1$) decision variables and Q objectives ($Q > 1$) subject to several constraints in Eq. (1).

$$\begin{aligned} &Minimize\ \ f(x) = [f_1(x), f_2(x), \ldots., f_Q(x)] \\ &S.t. \\ &\qquad x \in X \end{aligned} \tag{1}$$

Where $X \subseteq \mathbb{R}^Q$ is the feasible solution space and $X = \{X_1, X_2, \ldots, X_p\}$ is set of *p*-dimensional decision variables.

Nowadays, in most production centers, the need to answer the question of appointing a mixture of the production of commodities is felt more than ever before. In order to close the gap between the conditions of the problem and the real world conditions in this research, the multi-item lot-sizing problem has been studied with considerations of production line equilibrium limitation, and capacity limitation. Not only has there been a consideration of different production manners for products, but also the model has been designed in the conditions of having safety stock and shortage being allowed.

As best of our knowledge, three objective functions based on JIT concept are simultaneously considered to make the model near to reality. The main goal is to present a bi-objective mathematical model to optimize production, inventory, and shortage quantities as well as determine the best production manner in which summations of production, setup, inventory, and shortage costs as well as total storage cost are minimized. In order to formulate the mathematical model of the problem, the assumptions, parameters, decision variables, and mathematical formulation are provided as following subsections:

A. Assumptions

- The demand is deterministic.
- Shortage is both backorder and lost sale, proportionally.
- Shortage and inventory costs must be taken into consideration at the end.
- Storage capacity limitations are considered.
- Raw material resources are capacitated.
- The quantity of inventory and shortage at the beginning of the planning horizon is zero.
- The quantity of shortage at the end of the planning horizon is zero.

B. Parameters

T: Number of periods in the planning horizon; $t = 1, \ldots, T$

N: Number of items; $i = 1, \ldots, N$

J: Number of production manners; $j=1, \ldots, J$

d_{it} : The demand for item *i* in period *t*

φ_{it} : Unitary shortage cost of item *i* in period *t*

ξ : Probability of backorder shortage

φ_{it} : Unitary lost sale shortage cost of item *i* in period *t*

y_{it}^{-} : Unitary safety stock deficit cost of item *i* in period *t*

L_{it} : Safety stock value of item *i* at period *t*

δ_{it} : The safety stock variation between two consecutive periods

α_{ijt} : Unitary production cost of item *i* to production manner *j* in period *t*

β_{ijt} : The setup cost of item *i* to production manner *j* in period *t*

y_{it}^{+} : The unit holding cost of item *i* in period *t*

C_t : The amount of resource available in period *t*

v_i : The unit amount of resource necessary to produce item *i*

M: A large number

β_{ijt} : The loss setup cost of item *i* to production manner *j*

a_t : Unitary cost of storage space in period *t*

C. Decision variables

x_{ijt} : The quantity of item *i* produced to production manner *j* at period *t*

y_{ijt} : A binary variable equal to 1 if item *i* is produced to production manner *j* at period *t*

r_{it} : The shortage for item *i* at period *t*

s_{it}^{+} : Overstock deficit variables of item *i* at period *t*

s_{it}^{-} : Safety stock deficit variables of item *i* at period *t*

D. Proposed mathematical model

The mathematical model is as follows:

$$Min\ Z_1 = \sum_{i=1}^{n} \sum_{t=1}^{T} [(\sum_{j=1}^{J} (\alpha_{ijt}.x_{ijt} + \beta_{ijt}.y_{ijt})) + \xi.\varphi_{it}.r_{it} + (1-\xi).\pi_{it}.r_{it} + y_{it}^{+}.s_{it}^{+} + y_{it}^{-}.s_{it}^{-}] \qquad (1)$$

$$Min\ Z_2=\sum_{t=1}^{T-1}(x_{ij,t+1}-x_{ijt})^2 \qquad (2)$$

$$Min\ Z_3=\sum_{t=1}^{T}(x_{ijt}-d_{it})^2 \qquad (3)$$

S.t.

$$s^+_{i,t-1}-s^-_{i,t-1}-\alpha.r_{i,t-1}+r_{it}+\sum_{j=1}^{J}x_{ijt}=d_{it}+\delta_{it}+s^+_{it}-s^-_{it}$$
$$;\forall\ i=1,\ 2,\ ...,\ n\ ;\ t=1,\ 2,\ ...,\ T-1 \qquad (4)$$

$$s^+_{i,t-1}-s^-_{i,t-1}-\alpha.r_{i,t-1}+\sum_{j=1}^{J}x_{ijt}=d_{it}+\delta_{it}+s^+_{it}-s^-_{it}$$
$$;\ \forall\ i=1,\ 2,\ ...,\ n\ ;\ t=T \qquad (5)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{J}(v_i.x_{ijt}+f_{ij}.y_{ijt})\le C_t \qquad \forall\ t=1,\ 2,\ ...,\ T \qquad (6)$$

$$x_{ijt}\le M.y_{ijt}$$
$$;\forall\ i=1,\ 2,\ ...,\ n\ ;\ j=1,\ 2,\ ...,\ J\ ;\ t=1,\ 2,\ ...,\ T \qquad (7)$$

$$r_{it}\le d_{it} \qquad \forall\ i=1,\ 2,\ ...,\ n\ ;\ t=1,\ 2,\ ...,\ T \qquad (8)$$

$$s^-_{it}\le L_{it} \qquad \forall\ i=1,\ 2,\ ...,\ n\ ;\ t=1,\ 2,\ ...,\ T \qquad (9)$$

$$x_{ijt},r_{it},s^+_{it},s^-_{it}\ge 0\ \ ;y_{ijt}\in\{0,1\}$$
$$;\ \forall\ i=1,\ 2,\ ...,\ n\ ;\ j=1,\ 2,\ ...,\ J\ ;\ t=1,\ 2,\ ...,\ T \qquad (10)$$

As it can be observed above, the extended model is formulated. Objective (1) is the first objective function which is minimized total cost.

The objective function (1) minimizes the total cost including unit production costs with different production manner, inventory costs, overtime costs, shortage costs and setup costs.

The objective functions (2) and (3) minimize the variations of production levels.

Constraints (3) are the inventory balance through the planning horizon.

Constraints (4) the inventories balance at the end of period because the shortage is not permitted at the end period.

Constraints (5) are the capacity constraints; the overall consumption must remain lower than or equal to the available capacity.

Constraints (6) impose that the quantity produced must not exceed a maximum production level M_{it} which set to the minimum between the total demand requirement for item i on section $[t,T]$ of the horizon and the highest quantity of item i that could be produced regarding the Capacity constraints. M_{it} is then equal to

$$Min\ \left\{\sum_{t'=t}^{T}d_i t',(c_t-f_{it})/v_{it}\right\}$$

Constraints (7) and (8) define upper bounds on respectively, the demand shortage and the safety stock deficit for item i in period t. Finally, Constraints (10) enforces the restrictions on the decision variables.

III. THE MOSA ALGORITHM

As mentioned previously, solving the proposed non-linear integer programming model is difficult for obtaining a global optimal solution, so the use of meta-heuristic methods is very common. Another type of these methods is simulated annealing (SA) which was introduced by Kirkpatrick et al. (1983). The proposed MOSA algorithm is illustrated more in below subsections:

Since fitter presentations have higher probabilities of being selected; the algorithm converges to the best representation which expectantly indicates the optimum or near optimal solution to the problem after several generations. In the next subsections, we demonstrate the steps required to solve the model within MOSA framework. It is required to mention that a new representation structure is proposed to enhance being feasibility of each representation. In the below subsections, the required steps of our proposed MOSA are illustrated.

A. Initialization

The parameters of the proposed MOSA are included as follow: The inputs of the proposed MOSA are included as follow: (1) Initial temperature (T_0) which is a starting point for computing the temperature amount at each iteration (2) Population size (*nPop*) which is number of the solutions for sustaining in each generation. In fact, to increase getting to the better solution, we consider this parameter for the proposed MOSA. This policy causes to select best solution though population solutions; (3) Number of iteration in each temperature (*nIt*); (4) temperature reduction rate (β) which is calculated as Eq. (11).

$$T_h = \beta \times T_{h-1} \quad ; \quad h \succ 2, \;\; 0 \prec \beta \prec 1 \tag{11}$$

where T_h, β are the temperature at iteration h and the temperature reduction rate, respectively. In this research, we utilize the parameter tuning procedure to set the algorithm inputs as optimum state. Also, to generate initial population, random generation policy has been utilized.

B. Representation Structure

In order to increase feasibility of problem representations and satisfy more constraint, Fig. 1 presents the general form of a proposed problem representation.

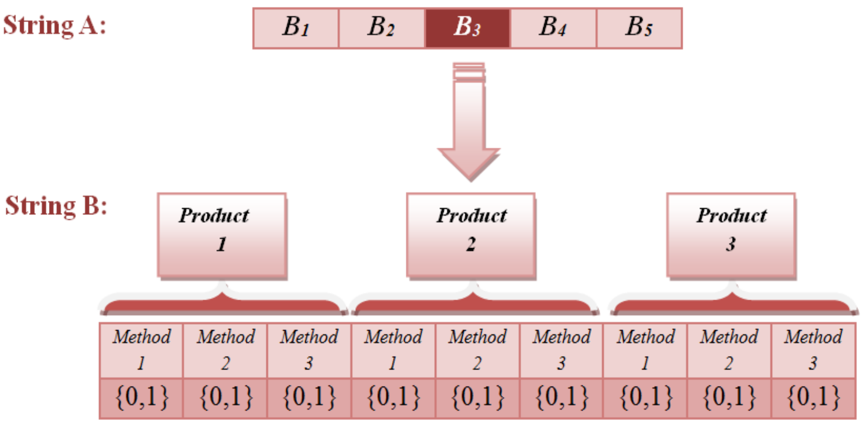

Fig. 1. Representation structure

C. Cost function evaluation

In order to evaluate each solution at each generation, we utilize objective function value. Each individual will be evaluated for the combined objective function of f_1, f_2. In order to do so, L_P-metric technique for transforming different objectives into one objective has been considered. In our investigation, the weights must be determined by decision maker (DM). Therefore, in order to authorize the weights selection by system provider and increase applicability of the proposed model, we choose a weighted method called L_P-metric $(P \in ([1,\infty) \cup \{\infty\})$ as our investigated technique. In this method, the difference between each objective function with its optimum value is minimized as Eq. (12).

$$\begin{aligned} &\textit{Minimize} \quad \left(\sum_{i=1}^{Q} \left[\eta_i \left| \frac{f_i^* - f_i(x)}{f_i^*} \right| \right]^P \right)^{1/P} \\ &\textit{Subject to:} \\ &\qquad x \in X \subset \mathbb{R}^Q \end{aligned} \tag{12}$$

Where η_i is the weight of objective function i which is determined by DM. Parameter Q indicates number of investigated objective functions. In particular, for $P = 1$, the definition yields the so called Manhattan metric, L_2 is the Euclidean metric, and L_∞ is the Tchebycheff metric. Since the structure of the problem can depend on the choice of the metric, we discuss on various types of P amount (Pasandideh, et al., 2013).

Besides, the most well known approach for handling constraints is penalty functions policy. So in order to control infeasible solutions, the penalty policy must be employed. In the proposed algorithm, the penalty is defined as a positive coefficient. The

penalty value will be considered zero, when problem representation is feasible and it will be selected as a non-zero value, even though one of the constraints is not be satisfied (Yeniay and Ankare, 2005).
When a problem representation is feasible, the penalty value will be zero and, otherwise, the penalty value will be multiplied to the cost function value.

4.2.1. Main loop of the MOSA

MOSA is a general random search algorithm which the solution area based on stochastic mechanism of physical annealing process in metallurgy is formed. Generally, the objective value of a solution is equivalent to the internal energy state. This algorithm starts with a high temperature and randomly chooses initial solution (ω_0). Also it should be mentioned that a primary value of T_0 act as a controller parameter of temperature. After that a new solution within the neighborhood of the current solution is calculated (ω_n). In fact, obtained solution is created in the neighborhood of the previous solution (ω). If the value of the combined objective function (f(ω_n)) is less than the previous value (f(ω)), the new solution is accepted (in minimization problems). Otherwise, in order to escape from the local optimal solution, the new solution will be accepted with a probability amount ($Probabilty_{MOSA}$) as Eq. (13). This process is repeated until the desired state of the algorithm is reached.

$$\text{Probability}_{MOSA} = e^{-\frac{\Delta}{T}} \quad ; \quad \Delta = \frac{f(\omega_n) - f(\omega\ )}{f(\omega_n)} \times 100 \qquad (13)$$

D.Design of Neighborhood representation

To indicate the neighborhood structure, the following procedure has been considered as Fig. 2 as similar as swap mutation (Radcliffe, 1991).

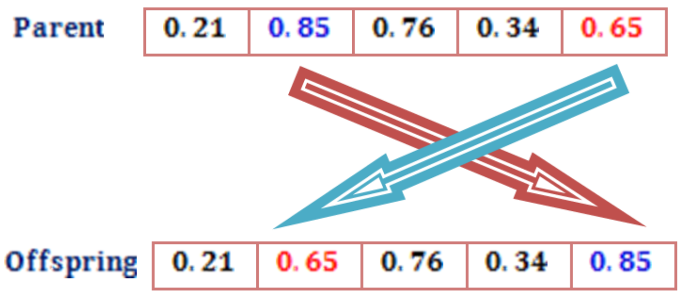

Fig. 2. An example of Neighborhood representation

E. Stopping criteria

The algorithm can also be stopped after a predetermined number of iterations.

IV. RESULTS

In order to evaluate the best performance of the solving methodology, several test problems are considered. Since the nature of this problem is NP-hard, the exact model is hard enough to select as a solving methodologies (Karimi et al., 2003). For such problems with regarding to the complicated calculation procedures and time consuming optimization models, the use of meta-heuristic algorithms is very common. The response variable is considered for selecting the best methodology.
The experiments are implemented on 10 problems. Then, these instance problems are totally solved. In fact, various problems with different dimensions are determined to obtain the best performance of the solving methodology. To do so, we run 40 problems for every algorithms and objectives which totally is to be equal 120 problems for all considered solving methodologies. Also, for achieving to the better solution and eliminate the uncertainty in the random generations, we run each problem three times. Then, averages of these three runs are computed and consider as ultimate solution. Also, it needs to be mentioned that, we report combined objective function as final objective function after running it again for each methodology. Table I provides text problems in order to demonstrate readability of the MOSA.

TABLE I: General data

Problem Number	Number of Items (*N*)	Number of Production Manners (*J*)	Number of Periods (*T*)
1	4	5	3
2	9	6	3
3	16	7	5

4	20	9	6
5	42	15	11
6	57	17	12
7	62	21	14
8	77	25	18
9	90	38	22
10	105	42	25

Implementing the proposed MOSA algorithm with the obtained values of problem number 8, after fifty generations, the algorithm has been converged with a combined cost function of 0.46096 in Fig. 3.

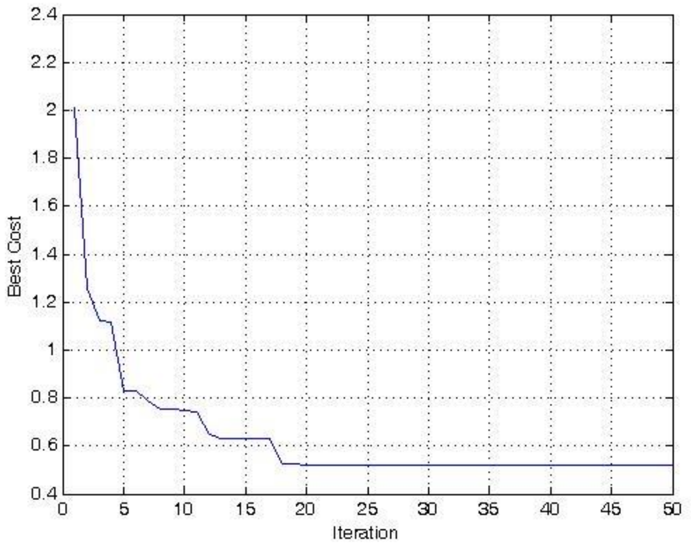

Fig. 3. Convergence diagram of problem No. 4

V. CONCLUSION AND FUTURE RESEARCH

In this paper, we have proposed a new multi-objective MILSP to determine optimal production quantity. Three objective functions are provided including minimizing total cost, and the objective in terms of JIT concept to balance the production process. Since the problem is NP-hard, an MOSA algorithm is proposed to solve the model. Finally, to demonstrate the efficiency of the proposed MOSA, we reported computational results on different sizes of the problems. As future research, one can present Pareto-based algorithms such as NSGA-II for comparisons.

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