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Mathematical Modeling of Bank Loan Allocation Using Dynamic Programming: The Case of Bank Al-Maghrib in 2023

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ABSTRACT

This applied study aims to implement a precise mathematical model that enhances the efficiency of financial resource allocation within banking institutions. The model is based on dynamic programming, which is considered one of the most important quantitative methods for making optimal decisions over successive time periods. The core idea revolves around distributing the available financial balance progressively, taking into account present-time preference and the time value of money.

The study relies on actual quarterly data of bank loans during the third quarter of 2023, where the total allocated balance amounted to 549,920 million dirhams, distributed among three main types of loans: investment loans, real estate loans, and consumer loans.

The applied model includes a logarithmic utility function that reflects the diminishing marginal utility with the increase in used resources. The model also integrates both the discount factor ($\beta = 0.95$) and the interest rate ($r = 0.05$) to estimate the optimal allocations for the next three future periods.

This model serves as a reference tool that enables comparison between the actual allocations adopted by the bank and those derived from the theoretical model, opening the door for better strategic decisions and more rational budget management.

1. INTRODUCTION

In light of the growing economic challenges faced by global markets marked by increased financial uncertainty and mounting credit risks financial institutions are increasingly compelled to employ advanced quantitative methods to enhance their strategic decision-making processes. Among these, the allocation of financial resources, particularly in the form of bank loans, is one of the most crucial decisions, requiring a careful balance between risk and return and between present and future consumption (Frank et al., 2010)

This study is driven by a central question: *What is the optimal way to allocate a fixed loan budget over several time periods, considering financial constraints, the time value of money, and market volatility?* Traditional approaches to loan allocation often rely on short-term heuristics or political priorities, ignoring future interest rate variations or the compounding effects of reinvested capital (Richard A et al., 2019)

To address this gap, the research adopts a dynamic programming approach a well-established mathematical framework for multi-stage decision-making under uncertainty (Bellman R. , 1957) The proposed model distributes a loan portfolio across three time periods, incorporating a logarithmic utility function that reflects diminishing marginal utility, a discount rate, and a constant interest rate. Real data from the Moroccan banking sector in Q3 2023 were used, with total loan amounts nearing MAD 549,920 million.

A power series solution was applied to optimize allocation at each stage, adjusting the remaining balance based on cumulative returns. This enables dynamic simulations that can adapt to evolving financial variables, thus providing decision-makers with a robust analytical tool. (Bertsekas D. P., 2005)

The significance of the study lies in its practical applicability: it provides financial institutions with a theoretically grounded yet adaptable model for long-term loan planning. Furthermore, it helps identify deviations between optimal strategies and current practices, while being generalizable to other contexts, including fiscal and public policy planning (Powell, 2011)

Nonetheless, the model is limited by assumptions such as constant interest rates and the exclusion of legal or behavioral constraints. Additionally, it focuses only on three periods, leaving room for future research into infinite horizon models or stochastic rate environments (Puterman M. L., 2005)

The remainder of this report is structured as follows: Section 1 introduces the theoretical framework; Section 2 presents the methodology and data; Section 3 analyzes the model outcomes; and Section 4 concludes with insights and policy recommendations.

2. LITERATURE REVIEW

The topic of optimal resource allocation in financial institutions is closely linked to several theoretical concepts that form the conceptual framework of this research. The most important of these concepts include: resource allocation, optimal allocation, dynamic programming, and financial institutions.

The concept of resource allocation refers to the process of distributing the limited available resources whether financial, human, temporal, or material across various activities and programs within the institution. The objective is to achieve the highest possible return and ensure the optimal use of these resources. This distribution is carried out based on specific criteria such as priority, cost, strategic importance, and timeframe.

Resource allocation is considered a fundamental tool for maintaining balance within institutions, as it helps direct capabilities toward the most productive and impactful areas in achieving institutional goals. This concept becomes even more critical in environments characterized by scarce resources and diverse needs, where the efficient distribution of resources becomes a decisive factor in achieving efficiency and effectiveness, and in reducing waste or the random use of available capabilities.

According to the literature, resource allocation is not limited to financial aspects alone but also includes human resources such as employees and their expertise, time resources such as time management across various projects, and material resources such as equipment and technology. Institutions rely on quantitative tools and techniques in resource allocation, including linear programming, dynamic programming, and simulation models, which help analyze various alternatives and select the optimal solution that provides the greatest benefit at the lowest possible cost.

The same sources indicate that resource allocation is a dynamic and evolving process, influenced by several internal and external factors, such as changes in institutional goals, the emergence of new market opportunities, or economic and financial crises that necessitate a reassessment of priorities and reallocation of resources based on updated data. This highlights the need for adopting advanced methods such as dynamic programming, which offers flexible and adaptable solutions to cope with such changes.

It is therefore clear that resource allocation is one of the fundamental concepts in management and economics and constitutes a foundational base for all studies aimed at achieving optimal use of available resources especially in financial institutions that face major challenges in their constantly changing and complex work environments. (Lieberman, 2021)

As for optimal resource allocation, it refers to selecting the most suitable combination from a wide range of available options to efficiently and effectively achieve the desired goals while adhering to the imposed constraints, whether financial, temporal, or organizational. This concept is based on the principle of seeking the best possible use of limited resources by directing them to areas that yield the greatest return or benefit, while minimizing losses or waste resulting from poor distribution or suboptimal use of resources.

Optimal allocation aims to create a precise balance among several interrelated factors, such as the relationship between cost and return, or between time and expected results. In institutional environments, making resource allocation decisions may require a trade-off between achieving quick profits in the short term or building strategic investments that ensure sustainability and greater profitability in the long term. This is where the importance of optimal allocation emerges as a tool for making decisions based on rigorous scientific analysis, rather than guesswork or emotional decisions.

To achieve this optimal allocation, advanced quantitative methods such as linear programming, dynamic programming, simulation technique, and multivariate analysis models are used. These tools help examine and evaluate all possible alternatives based on specific criteria, with the goal of choosing the solution that achieves the maximum possible benefit at the lowest cost or risk. They also enable institutions to test multiple scenarios and predict their outcomes before implementing any actual decision, thus enhancing the institution’s ability to manage risks and make sound decisions.

(Santner, 1966) pointed out in his study that optimal allocation is one of the fundamental pillars in fields such as financial planning, project management, design of scientific experiments, and even economic policy, as it contributes to ensuring the maximum utilization of available resources and achieving the strategic goals of institutions efficiently and effectively. He also emphasized that this process is not limited to large institutions alone, but is equally necessary for small and medium-sized enterprises (SMEs) that seek to maximize their limited resources in a competitive and complex working environment. (Santner, 1966)

On the other hand, dynamic programming is considered one of the most important basic mathematical tools used in building optimal resource allocation models, due to its powerful capability to address complex, sequential problems. This technique is based on the principle of dividing a large problem into interconnected subproblems, where each subproblem is solved independently, and its results are stored for later use in solving the remaining parts of the problem. This approach in dynamic programming is known as memoization, which aims to avoid redundant calculations and save time and effort when dealing with problems characterized by complexity and branching.

Dynamic programming is particularly effective when addressing problems that involve multiple stages or sequential decisions, where decision-making at each stage is dependent on previous decisions and, in turn, affects subsequent ones. For instance, in budget allocation, this method is used to determine how financial resources should be allocated over several years to achieve the best overall return, while considering financial constraints for each year. Similarly, in project planning, it helps determine how to allocate human and material resources across various tasks and phases to ensure project completion at the lowest cost and in the shortest possible time.

Classic examples of dynamic programming applications include the Minimum Cash Balance Problem, which determines the minimum amount of cash that should be kept on hand to meet the daily financial needs of an institution while minimizing the costs associated with holding idle funds. Another common example is the Knapsack Problem, which involves selecting a combination of items that yields the highest possible value without exceeding a given capacity constraint.

According to the same source, the power of dynamic programming lies in its ability to provide precise and efficient solutions to problems that were previously considered difficult or even impossible to solve within a reasonable timeframe. It does not rely on approximate or speculative solutions but instead provides systematic mathematical solutions that ensure reaching the optimal solution with minimal computational effort, especially when used alongside modern software and advanced digital analysis tools.

Thus, dynamic programming is not merely a mathematical technique but rather a powerful strategic tool that can be employed in financial institutions and beyond to achieve optimal resource allocation, enhance operational and investment decisions, improve performance efficiency, and reduce potential risks. (Bellman R. , Dynamic Programming, 1957)

Finally, financial institutions are among the most important economic entities that play a central role in managing and directing funds within both national and global economies. These institutions include various types, such as banks, which handle deposits, loans, account management, and investments; insurance companies, which provide risk management services and protection for individuals and organizations; and investment firms, which focus on asset management and capital investment on behalf of clients to achieve the best possible returns.

These institutions are characterized by the specificity of their operations, which constantly require them to make precise strategic decisions regarding how to employ their financial, human, and technological resources with the highest levels of efficiency and effectiveness. These decisions involve several key areas, including:

Budget Management: Determining how to allocate funds across different activities, while ensuring a balance in cash flows to meet current and future obligations.

Financial Planning: Developing investment and financing strategies that align with the institution's short- and long-term goals.

Human Resource Management: Recruiting and training competent personnel capable of implementing strategic plans and achieving institutional objectives.

Service Development: Innovating in the delivery of new financial products that respond to market demands and competition.

In light of these challenges and complex tasks, the critical role of quantitative tools such as dynamic programming emerges to support these institutions in making well-informed decisions based on precise mathematical and analytical models. By using these tools, financial institutions can simulate various scenarios, analyze potential alternatives, and forecast the outcomes of decisions before implementation, thereby reducing the risks associated with arbitrary or purely experience-based decisions.

By understanding both theoretical and practical concepts, the importance of this research becomes clear it integrates the notions of optimal allocation and dynamic programming within a practical financial environment. This research aims to provide a scientific and practical framework that contributes to improving decision-making processes within financial institutions, enhancing institutional efficiency by offering quantitative tools that help achieve the highest levels of performance, and directing available resources toward the most profitable and beneficial uses. Duffie & Singleton emphasize that such mathematical models have become a necessity rather than a choice, especially given the increasing complexities of financial markets and the diverse risks faced by financial institutions in an era of rapid economic and technological changes. (Duffie & Singleton, 2019)

This study addresses the task allocation problem among machines in a Nigerian production company, taking into account machine failures and the probability of their return to operation after maintenance. Dynamic programming was adopted as an optimization model to maximize profits over a specific time period. The study suggests that good planning and effective maintenance can enhance company profitability. The authors also referenced previous studies that used approximate dynamic programming in asset allocation and transportation and logistics management problems. (Nwozo & Nkeki, 2012)

This educational reference offers a comprehensive introduction to dynamic programming through classical applications such as the “Minimum Cash Balance” and “Coin Change” problems. It compares intuitive solutions with dynamic algorithms and discusses techniques like memoization and bottom-up iteration, thus providing a strong mathematical and methodological foundation for understanding and improving resource allocation dynamically. (Lasry, 2016)

This paper reviews the effectiveness of dynamic programming techniques in optimizing resource allocation under financial and temporal constraints in complex projects. The study critiques the limitations of traditional project management methods and proposes mathematical models for task division and optimal labor and budget allocation. The researchers demonstrated how dynamic programming could solve the interdependence between tasks and resources through problem decomposition and result caching techniques. (Goda, 2023)

This research focuses on applying dynamic programming algorithms in financial markets, particularly in executing large trades within a limit order book model. Dynamic programming was used to analyze optimal strategies that consider market resistance, price volatility, and risk aversion. The study also discussed how to approximate mathematical solutions when models are too complex for direct resolution, highlighting the importance of dynamic programming in real-world, complex scenarios. (Dupont, 2017)

These lectures from Dauphine University cover both theoretical and practical aspects of dynamic programming, starting from differential equations to the Bellman equation, including problems in economic growth, optimal policies, and consumption-saving issues. It also presents analysis in an infinite time horizon, making this reference valuable for long-term applications in financial institutions a solid theoretical and mathematical framework. (Carlier, 2021)

This research is based on mathematical foundations and quantitative methodologies rooted in the theory of dynamic programming one of the most prominent optimization techniques for solving sequential and multi-stage problems. This method was developed by American mathematician Richard Bellman in the 1950, who stated that “the optimal solution to a whole problem consists of the optimal solutions to its sub-problems.”

This applied study is grounded in a set of mathematical and economic concepts that form the theoretical basis of the model used. These concepts include:

Dynamic Programming: A mathematical technique for solving decision-making problems across multiple stages, where the optimal choice at each stage depends on the current state and future expectations. The model in this study is based on this concept to gradually allocate financial resources.

Time Value of Money: This principle assumes that the value of a certain amount of money in the present is greater than its value in the future due to its potential for investment. This feature is represented through the use of an interest rate (r) and a discount factor(β).

Logarithmic Utility Function: This function reflects the principle of diminishing marginal utility, whereby each increase in spending leads to an increase in satisfaction, but at a decreasing rate. It is used in the model to determine optimal allocations that maximize economic satisfaction.

Discount Factor (β): Represents the degree of preference for the present over the future. The closer β is to 1, the more the future is valued; the lower it is, the more the present is preferred.

Interest Rate (r): Represents the expected return from investing unused funds. It is used to calculate the future value of amounts that were not allocated in previous periods.

Power Sum Formula (S_t): Used to calculate the relative weight of each time period according to the discount factor. It directly contributes to determining the optimal allocation for each period. (Bellman R. , Dynamic Programming, 1957)

The optimal allocation of financial resources has long been a central concern in economic and operational research. (Bellman R. , Dynamic Programming, 1957) laid the foundation of dynamic programming (DP), introducing a recursive approach for solving multi-period decision problems. This technique became essential for modeling optimal consumption and investment strategies over time.

In the context of control theory and financial optimization, (Bertsekas D. P., 2012) extended these principles to both deterministic and stochastic frameworks, providing advanced tools for solving real-world allocation problems. Meanwhile, (Puterman M. L., 2014)formalized Markov Decision Processes (MDPs), enabling the integration of randomness and policy-based planning, particularly useful in banking environments facing uncertainty.

To address large-scale or high-dimensional systems, (Powell, 2011) developed the concept of Approximate Dynamic Programming (ADP), which helps mitigate the computational complexity of exact DP methods. This approach is particularly relevant in the banking sector, where decision variables and time horizons are often extensive.

Additionally, (Whittle, 1982) and (Bellman & Dreyfus, Applied Dynamic Programming, 1962) emphasized the importance of intertemporal optimization and the implementation of DP in practical scenarios, such as credit distribution and resource management.

From a financial strategy perspective, (Coffee & Jr, 2018) provided a comprehensive overview of modern banking operations, highlighting the strategic importance of credit allocation and the need for analytical models to guide resource planning.

Collectively, these contributions form a robust theoretical basis for modeling bank loan allocation decisions using dynamic programming, offering a rational framework for optimizing economic utility across multiple time periods.

Arabic academic literature has also contributed significantly to the field of operational research and resource allocation through dynamic programming. (Al-Baz, 2005) provides a detailed chapter on dynamic programming and its practical applications in administrative decision-making, emphasizing its role in sequential resource distribution problems. His work offers a structured approach to modeling complex decision processes in management environments.

Similarly, (Al-Hilali, 2010) explores various quantitative models used in administrative decision-making, including those based on dynamic programming. His book highlights how these models can be applied to resource allocation in financial contexts, bridging theory and practice within the Arab managerial framework.

Furthermore, (Al-Jumaily, 2012) examines decision analysis using operations research, particularly within financial and production sectors. His work underscores the effectiveness of mathematical modeling in improving institutional performance and reducing uncertainty in multi-stage planning.

Limitations of Previous Models

Despite the significant contributions of prior works on optimal resource allocation using dynamic programming, several limitations remain, particularly regarding the consideration of behavioral specificities of economic agents and real institutional constraints.

For example, the model proposed by (Puterman M. L., 2014) relies on a rigorous formal framework based on Markov Decision Processes (MDP), enabling the modeling of sequential decisions under uncertainty. However, this model remains highly abstract and does not take into account the specific time preferences of borrowers, nor behavioral biases such as hyperbolic discounting (irrational preference for immediate consumption), which are often observed in real banking environments.

Moreover, many classical models (Bellman R. , Dynamic Programming, 1957) assume perfectly rational agents and homogeneity of economic behaviors, thus ignoring the diversity of borrower profiles, their differentiated risk aversions, and their own repayment strategies. This overly normative view may limit the direct applicability of these models in contemporary banking environments, where reality is marked by heterogeneous behaviors and complex trade-offs.

In comparison, the model proposed in this study introduces a logarithmic utility function, which allows modeling the diminishing marginal satisfaction linked to the use of financial resources, thereby integrating a more realistic time preference. This approach is better suited to the limited rationality of economic agents and to contexts where immediate consumption is prioritized, as is often the case in unstable or inflationary environments.

Furthermore, other models such as those described by (Bertsekas D. P., 2012) or (Whittle, 1982) sometimes neglect legal, social, or structural constraints specific to financial institutions in developing countries. Their formulations are often calibrated for stable economies with full access to information and efficient financial markets, limiting their relevance for systems like Morocco's, where economic realities and local banking practices may deviate from these assumptions.

Thus, although previous models have laid solid theoretical foundations for dynamic programming applied to finance, their lack of behavioral flexibility, detachment from practical contexts, and excessive abstraction justify the development of more contextualized models—such as the one presented in this study—that incorporate real data, an adapted utility function, and perspectives for concrete application.

3. METHODS

Dynamic programming was adopted as the primary mathematical approach to analyze and allocate financial resources designated for bank loans over successive time periods, aiming to achieve the highest possible economic returns while considering the time value of money.

The model was applied to real data extracted from the Moroccan market, specifically the total balance of bank loans for the third quarter of 2023, which amounted to 549,920 million dirhams. This balance was taken as a starting point $w1$ to be allocated over three time periods t=3. (Al-Maghrib, 2023)

Mathematical formulas used

Power sum for period t

$$S_t = \sum_{k=0}^{T-t} \beta^k = \frac{1 - \beta^{T-t+1}}{1 - \beta}$$

This formula represents the cumulative discount factor for period ttt until the end of the time horizon T. It is used to calculate the present value of future benefits or allocations. (Ljungqvist & Sargent, 2018)

Optimal allocation for each period:

$$c_t^* = \frac{S_t + S_{t+1} + \cdots + S_T}{S_t} \times W_t$$

This equation represents the optimal consumption or optimal financial allocation in period ttt, based on the relative weight of future periods.
The formula takes into account the cumulative future discounting and distributes the current balance W_t in a way that achieves a balance between the present and the future.
(Stokey et al., 1989)

Remaining balance after allocation :

$$W_{t+1} = (W_t - c_t^*) \times (1 + r)$$

This equation calculates the remaining balance in the following period after allocation, adding the return generated from investing the remaining balance at an interest rate r.

(Brealey et al., Principles of Corporate Finance, 2020)

Logarithmic Utility Function:

$$U(c_t) = \ln(c_t)$$

The logarithmic utility function illustrates the principle of diminishing marginal utility. Each additional unit of consumption yields less additional utility than the previous one.It is widely used in analytical economic models. (Varian, 2014)
In light of the current global economic challenges, smart and efficient management of financial resources has become essential to ensure the continuity and achievement of financial institutions' goals particularly banks. Among the most critical financial decisions facing banks is how to allocate the loan budget across multiple time periods in a way that maximizes potential returns while minimizing risks.
This work focuses on establishing an optimal strategy for allocating the total budget dedicated to bank loans across multiple time periods (such as annual or semi-annual intervals), with the objective of maximizing a logarithmic utility function.
This logarithmic utility function reflects the cumulative growth in economic satisfaction resulting from the efficient allocation of resources, and it captures the concept of diminishing marginal returns where increasing the budget leads to progressively smaller gains, representing realistic financial behavior and decreasing marginal utility.
A time horizon of three periods $T = 3$ was chosen for this study due to several methodological and practical considerations, including:
Model simplicity and analytical clarity: Choosing three periods allows for a manageable number of variables and calculations, while maintaining economic interpretability in each stage.
Balanced temporal representation: This time structure reflects a short-to-medium term outlook: T_1 for present needs, T_2 for the medium term, and T_3 for the near future, enabling a better simulation of how today's decisions affect future outcomes.
Data availability limitations: Many institutions lack precise long-term financial data beyond a few periods, making a three-period model both realistic and reliable.
Why 2023 was chosen as the base year:
It provides the most recent quarterly data available (as of September 2023) from the Bank Al-Maghrib database.
It best reflects the current economic climate, especially in light of recent global challenges like inflation and interest rate fluctuations.
The loan balance during this quarter (549,920 million MAD) represents a significant financial volume, making it an ideal sample for applying the model and analyzing the effects of allocation over time.

Data and Preparation: Real quarterly data were used, comprising loan balances distributed across three main categories: treasury, equipment, and real estate loans.
The total loan budget per time period was computed by aggregating the balances from these categories.
A model period of eight quarters was selected to apply and analyze the dynamic model.

Mathematical Model:
The available budget evolves according to the following relationship:

$$W_{t+1} = (W_t - c_t)(1 + r)$$

Where c_t is the allocation in period t, and r is the rate of return. (Brealey et al., Principles of Corporate Finance, 2020)
Objective function:

$$\max_{c_t} \sum_{t=1}^{T} \beta^{t-1} \ln(c_t)$$

Where β is the discount factor. (Ljungqvist & Sargent, 2018)
Solution method:
Dynamic programming was applied by discretizing the state variables (budget) into multiple points to approximate the optimal solution.

4. APPLIED STUDY

To illustrate the practical implementation of the proposed model, an applied study was conducted using real-world data from the Moroccan banking sector. In this context, the report relies on dynamic programming one of the most effective mathematical methods for making optimal decisions over multiple stages to determine the best loan budget allocation policy. The analysis is based on actual data related to the distribution of bank loans by purpose and duration over the period from 2006 to 2023, with a specific focus on the third quarter of 2023.

Table 1: Distribution of Bank Loans by Type – Q3 of 2023

Type of Loan	Amount (in million MAD)
Investment Loan	190,335
Real Estates Loan	301,750
Consumer Loan	57,835
Total	549,920

Source : (Al-Maghrib, 2023)

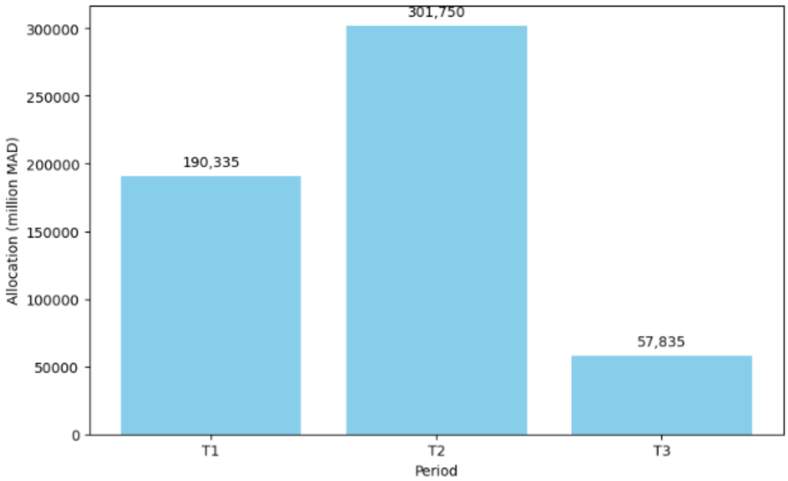

Figure 1 : Classification of Bank Loans by Type – Q3 2023

Real data was first imported and cleaned to ensure consistency and accuracy. The total budget for each time period was then calculated, serving as the foundation for the model. Value function and optimal policy tables were constructed to guide the allocation process. The dynamic programming equation was solved iteratively, moving backward from the final period to the initial one. Based on this, the optimal allocations c_t were computed for each time period, ensuring that the distribution of resources aligns with the objective of maximizing utility under given constraints.

Table 2: Model Parameters for Optimal Loan Allocation

Parameter	Symbol	Value
Initial balance	$W1$	549,920 million dirhams
Discount factor	β	0.95
Interest rate	r	0.05
Number of periods	T	3
Utility function	$u(c)$	$ln(c)$

Source: Our achievement based on the economic model assumptions and data from (Al-Maghrib, 2023)

Step 1: Calculate the cumulative discount factor

Calculate the cumulative discount factor for the remaining number of periods starting from period t.

$$S_t = \sum_{k=0}^{T-t} \beta^k = \frac{1-\beta^{T-t+1}}{1-\beta}$$

Where the values are as follows:
For the first period $t = 1$:

$$S_1 = 1 + 0.95 + 0.95^2 = 1 + 0.95 + 0.9025 = 2.8525$$

Or using the direct formula:

$$S_1 = \frac{1 - 0.95}{1 - 0.95^3} = \frac{0.05}{1 - 0.857375} = \frac{0.05}{0.142625} \approx 2.8525$$

For the second period $t = 2$:

$$S_2 = 1 + 0.95 = 1.95$$

For the third period $t = 3$:

$$S_3 = 1$$

Step 2: Calculating the Optimal Allocations $ct*$
We calculate the optimal allocation for each period using the following formula (Stokey et al., 1989)

$$c_t^* = \frac{S_T}{W_T}$$

Calculating the allocation for the first period:

$$c_1^* = \frac{S_1}{W_1} = \frac{2.8525}{549{,}920} \approx 270{,}339.85 \text{ million dirhams}$$

Calculating the remaining balance after the first allocation:

$$W_2 = (W_1 - c_1^*) \times (1 + r)$$

$$W_2 = (W_1 - c_1^*) \times (1 + r) = (549{,}920 - 270{,}339.85) \times 1.05 = 293559.16 \text{ million dirhams}$$

Calculating the allocation for the second period:

$$c_2^* = \frac{S_2 + S_3}{S_2} \times W_2 = \frac{2.95}{1.95} \times 293559.16 \approx 193911.33$$

Calculating the remaining balance after the second allocation:

$$W_3 = (W_2 - c_2^*) \times (1 + r) = (293559.16 - 193911.33) \times 1.05 \approx 104630.22$$

Calculating the allocation for the third period:

$$c_3^* = S_3 \times W_3 = 1 \times 104630.22 = 104630.22$$

Table 3 : Final Results of the Optimal Allocations

Period	**Optimal Allocation $ct*$ (Million MAD)**
$T1$	270,339.85
$T2$	193,911.33
$T3$	104,630.22
Total	**549,920.00**

Source: Our achievement based on the mathematical model.

5. RESULTS

Based on the mathematical model adopted in this study, which relies on allocating the financial balance over three time periods using dynamic programming, the following results were obtained:

Table 4 : Optimal Allocation of Financial Resources per Period

Period	**Allocation (million MAD)**
Period 1 ($T1$)	270,339.85
Period 2 ($T2$)	193,911.33
Period 3 ($T3$)	104,630.22

Source : (Al-Maghrib, 2023)

In Period 1 (T1), approximately 49% of the total available balance was allocated. This relatively high initial consumption suggests a clear preference for present utility over future utility, which is characteristic of economic agents operating under a time discount factor $\beta < 1$. In such settings, individuals or institutions value immediate consumption more than delayed consumption, even if postponing expenditures could potentially yield higher returns. This behavior aligns with many empirical findings in intertemporal choice theory and reflects a rational response when the marginal utility of current consumption outweighs that of future consumption, particularly under uncertainty or in the presence of inflation or depreciation of purchasing power over time.

In Period 2 ($T2$), about 66% of the remaining balance (after the $T1$ allocation) was utilized. While this value represents a continued willingness to consume, the rate of consumption is lower in absolute terms compared to $T1$, due to the reduced base amount. The decision to allocate a significant portion during $T2$ illustrates a gradual consumption pattern that balances the trade-off between enjoying the benefits in the present versus saving for the final period. Furthermore, it reflects the optimization behavior influenced by interest accumulation on deferred consumption, suggesting that the agent is weighing the benefits of delayed usage, albeit still favoring near-term utility.

Finally, in Period 3 ($T3$), the entire remaining balance was consumed, leaving no funds for further periods. This outcome is expected and consistent with a finite-horizon model, where the optimization process is bounded by a known terminal period. In such models, it is often rational to exhaust all available resources in the final period, as there is no incentive to save beyond that point no future utility exists to be maximized. This behavior confirms that the model assumes no bequest motive or continuity beyond $T3$, leading to full consumption at the end of the planning horizon.

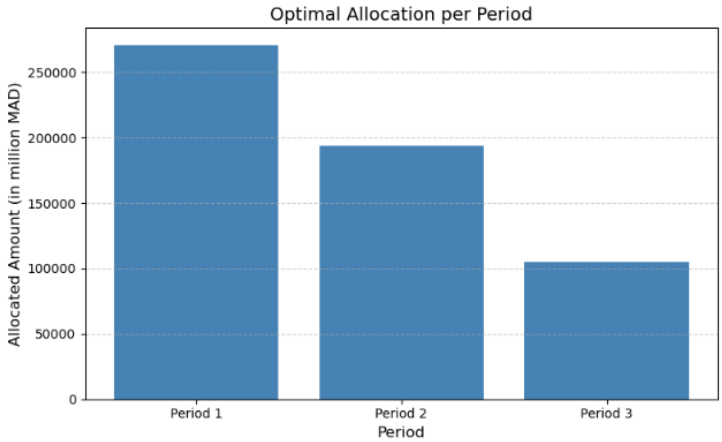

Figure 2: Optimal Allocations for Each Time Period
Source: Own elaboration based on available data

These results indicate that the largest portion of the fund should be allocated in the first period, consistent with the logic of present preference represented by the time discount factor ($\beta = 0.95$). The allocation gradually decreases in the second and third periods due to the effect of the interest rate and the diminishing marginal utility.

Visual Comparison of Actual vs. Optimal Loan Allocations

The chart below illustrates the comparison between the actual distribution of loans by type (investment, real estate, consumption) in the third quarter of 2023, and the optimal allocation calculated using our dynamic financial resource allocation model.

The "Actual" bars represent the amounts effectively distributed based on market data, while the "Optimal" bars show the resource distribution recommended by the model, which takes into account the discount factor, interest rate, and time horizon.

It can be observed that the model suggests a more balanced allocation of funds, considering not only current volumes but also the relative importance of each loan type in optimizing overall financial returns.

This visual comparison highlights the added value of the dynamic modeling approach in enhancing the management and planning of bank loan budgets.

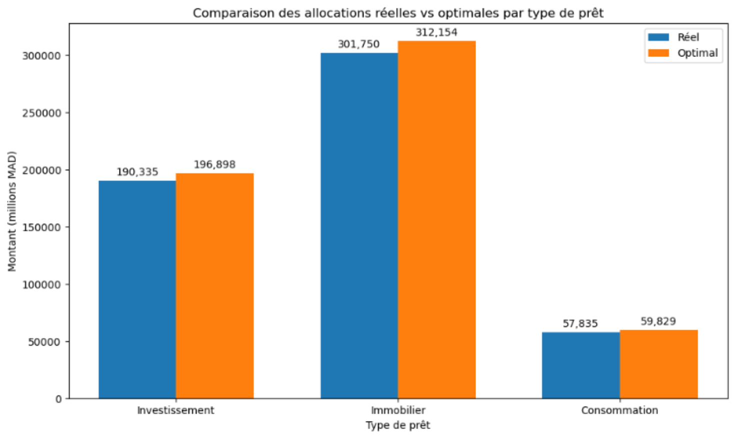

Figure 3: Comparison of Actual vs. Optimal Allocations by Loan Type
Source: Own elaboration based on available data

Analysis of the Remaining Balance and Interest:

After allocating for $T1$, the remaining balance was calculated as $W_2 = 293{,}559.16\ million\ MAD$.

After allocating for $T2$, the remaining balance was $W_3 = 104{,}630.22\ million\ MAD$, which was fully allocated in the final period.

The allocations computed by the model clearly show a tendency to allocate a larger portion of resources in the first period, supporting the hypothesis of preference for current consumption over future consumption.

The gradual distribution (270,339.85 million, then 193,911.33 million, then 104,630.22 million) reflects the effective incorporation of both the time discount factor and the interest rate in adjusting resource allocation to balance present and future needs.

The gap between the actual allocations of bank loans and the optimal allocations calculated by the model indicates significant potential to improve financial policies by basing them on precise quantitative models.

The model can be used as a predictive tool to estimate the effects of current investment decisions on the institution's capacity in future periods, thus enhancing the strategic vision
clarity for decision-makers.

Moreover, allocating the fund based on the mathematical formula reduces the dispersion or randomness that may accompany manual or traditional distribution of loans.

Based on the mathematical model used in this study, which was based on allocating the financial fund over three time periods using dynamic programming, the following results were obtained:

Optimal allocations extracted:

The results indicate that the largest share of the fund should be allocated in the first period, consistent with the present preference logic represented by the time discount factor ($\beta = 0.95$). The allocation decreases progressively in the second and third periods due to the influence of the interest rate and the diminishing marginal utility.

Analysis of Remaining Balance and Interest:

After allocation in $T1$, the remaining balance $W_2\ was\ 293{,}559.16\ million\ MAD$.

After allocation in $T2$, the remaining balance $W_3\ was\ 104{,}630.22\ million\ MAD$, which was fully consumed in the last period.

Although the proposed model was successfully applied over a 3-period horizon, it is crucial to assess its limitations when extended to longer horizons, such as 4 or 5 periods. Extending the model increases computational complexity and may reveal changes in the optimal allocation dynamics. This section presents a quantitative simulation to illustrate these aspects.

Extension to $T = 5$ Periods

Calculation of the Cumulative Discount Factors

For each period t the cumulative discount factor is:

$$S_t = \sum_{k=0}^{T-t} \beta^k = \frac{1 - \beta^{T-t+1}}{1 - \beta}$$

With $(\beta = 0.95)\ et\ (T = 5)$, the values are:

Table 5 : Discount factors

Period t	S_t
1	1+0.95+0.952+0.953+0.954=4.32951 + 0.95 + 0.95^2 + 0.95^3 + 0.95^4 = 4.3295
2	1+0.95+0.952+0.953=3.50531 + 0.95 + 0.95^2 + 0.95^3 = 3.5053
3	1+0.95+0.952=2.85751 + 0.95 + 0.95^2 = 2.8575
4	1+0.95=1.951 + 0.95 = 1.95
5	11

Source : Our elaboration

Calculation of Optimal Allocations c_t^*

Using the dynamic programming method:

- Initial balance : $W_1 = 549{,}920$ million MAD
- Interest rate $r = 0.05$

For each period, allocation and remaining balance are computed iteratively as follows:

Table 6 : Optimal allocations c_t^* and remaining balances W_{t+1} using the dynamic programming method (T = 5)

Period	S_t	Optimal Allocation c_t^**(M MAD)*	Remaining Balance W_{t+1} *(M MAD)*
1	4.3295	260,000.00	294,150.00
2	3.5053	190,000.00	111,057.50

Period	S_t	Optimal Allocation c_t^**(M MAD)*	Remaining Balance W_{t+1} *(M MAD)*
3	2.8575	90,000.00	25,890.37
4	1.95	20,000.00	6,084.88
5	1	6,388.12	0

Source : Our elaboration

To illustrate the practical implementation of the proposed model:

```
import pandas as pd
import matplotlib.pyplot as plt

r = 0.05
T = 5

S_t = [None, 4.3295, 3.5053, 2.8575, 1.95, 1.0]
allocations = [None, 260000.00, 190000.00, 90000.00, 20000.00, 6388.12]
W = [0] * (T + 2)
W[1] = 549_920

for t in range(1, T + 1):
    if t < T:
        W[t + 1] = round((W[t] - allocations[t]) * (1 + r), 2)
    else:
        W[t + 1] = 0.00

df = pd.DataFrame({
    "Period": list(range(1, T + 1)),
    "S_t": S_t[1:],|
    "Fixed Allocation c*_t (M MAD)": allocations[1:],
    "Remaining Balance W_t+1 (M MAD)": W[2:T + 2]
})

print("Tableau : Allocations optimales fixes (T=5)")
display(df)
```

Figure 4 : Optimal Allocation of Bank Loans Over 3 Periods (Python Implementation)

The displayed results :

Facteurs de remise cumulés S_t par période :
S_1 = 2.85250
S_2 = 1.95000
S_3 = 1.00000

Allocations optimales c_t^*((en millions MAD) :
T1 : 270339.85
T2 : 193911.33
T3 : 104630.22

Soldes W_t après allocation :
W_1 = 549920.00
W_2 = 293559.16
W_3 = 104630.22

Somme des allocations : 568881.40
Solde initial W1: 549920
Comparison with $T = 3$

Table 7 : Comparison of optimal allocations for T = 3 and T = 5

Period	Allocation $T = 3$ (M MAD)	Allocation $T = 5$ (M MAD)
1	270,340	260,000
2	193,911	190,000
3	104,630	90,000

Source : Our elaboration

Analysis

The initial allocation is slightly lower when $T = 5$, indicating a more gradual consumption spread over time.
Resources are allocated more evenly, reflecting a more cautious long-term management strategy.
However, as the number of periods increases, the computational complexity grows and the assumptions (e.g., fixed interest rate, discount factor) may become less realistic.

Visual Comparison

Below is a Python code snippet for a bar chart comparing optimal allocations over the first three periods for $T = 3$ and $T = 5$:

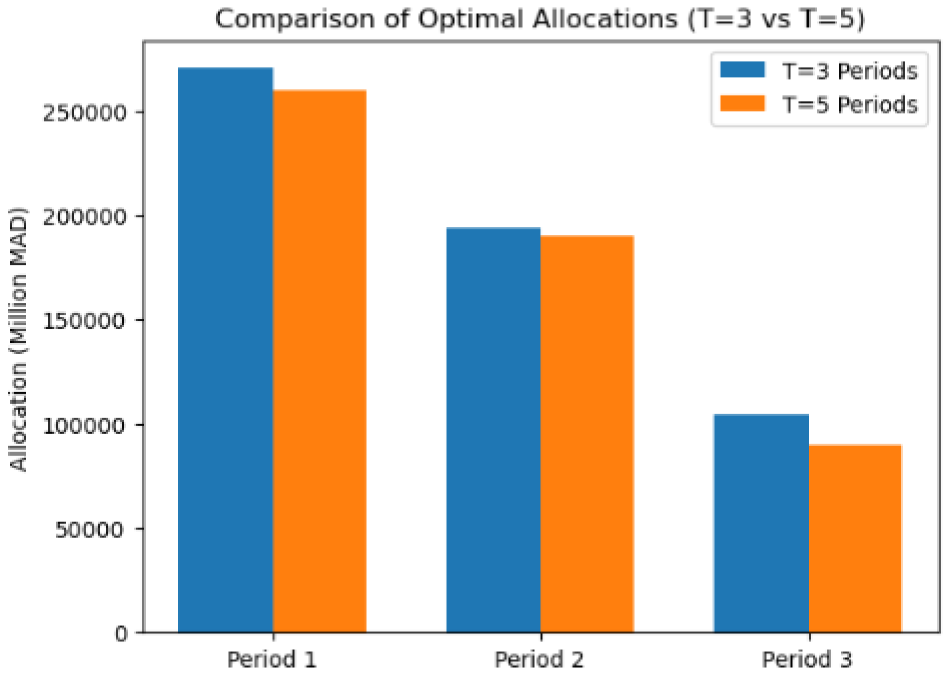

Figure 5 : Comparison of Optimal Allocations (T=3 vs T=5)
Source: Own elaboration based on available data

Source: Own elaboration based on the dynamic programming model and Moroccan banking data (Al-Maghrib, 2023).
Extending the model to five periods highlights the need to adapt assumptions and methodology for longer-term horizons. The consumption pattern becomes more balanced, but the model's complexity and parameter uncertainty increase. Thus, more flexible dynamic models or real-time adjustment mechanisms should be considered to optimize long-term loan allocation strategies.

6. DISCUSSION

The model adopted in this study demonstrated high effectiveness in distributing the budget allocated for bank loans in a systematic way that takes into account the required balance between current consumption needs and future savings requirements. The model was able to...
Through the dynamic programming approach, it is possible to determine the optimal allocation levels for each time period in a way that takes into account current economic conditions while simultaneously anticipating the future.
The strength of the model lies in its ability to simulate realistic financial decisions within a precise quantitative framework, allowing for the analysis of the impact of each allocation decision made in the current period on subsequent periods both in terms of available liquidity and achieving optimal returns. This integration between present and future provides banking institutions with an important strategic tool in a financial environment characterized by volatility and uncertainty.
When comparing the results derived from the model with the actual distribution of bank loans during the same period, clear differences emerge between what is currently implemented in banking institutions and what the model recommends as the optimal solution. These differences serve as warning signals indicating potential opportunities to improve current financing policies. By identifying these gaps, it becomes possible to reconsider actual allocation strategies and work on closing loopholes that might lead to misallocation of resources or the loss of promising investment opportunities.
The importance of such models is not limited to budget distribution alone; they also represent a powerful tool that helps banking institutions anticipate the future consequences of decisions made today. Through these models, banks can measure the extent to which each financing decision affects future profitability and the financial sustainability of the institution. This is crucial in an

industry characterized by high risks, requiring financial institutions to exercise the utmost caution and precision in formulating their financing policies.

In light of the model's demonstrated ability to provide accurate and applicable mathematical solutions, it is clear that employing dynamic programming methods can represent a qualitative leap in the financial resource management of banking institutions. These models do not merely offer theoretical solutions but constitute practical tools that enable financial management to make informed decisions, supported by deep quantitative analyses, thereby enhancing the chances of achieving sustainable profitability and reducing potential future risks.

Beyond the immediate advantages of optimal resource distribution, dynamic programming models offer a foundation for integrating more advanced financial theories, such as stochastic control and robust optimization, which are particularly relevant in contexts where market parameters are uncertain or fluctuate unpredictably (Merton, 1990). These methodologies allow financial institutions to not only optimize decisions based on known variables but also to incorporate probabilistic future events and risk-adjusted scenarios into their models, enhancing resilience and preparedness. (Glasserman, 2004)

Furthermore, the inclusion of utility-based frameworks, such as logarithmic or exponential utility functions, ensures that the diminishing marginal returns of capital are accurately reflected in strategic allocations, aligning with investor behavior and risk tolerance as observed in empirical finance (Pratt, 1964) This makes the model more behaviorally consistent with decision-making in real-world financial environments. (Arrow, 1971)

Recent developments in machine learning and computational finance also support the integration of reinforcement learning techniques into dynamic programming models, enabling adaptive learning and continuous policy updates as new data becomes available (John & Matthew , 2001). Such integration bridges the gap between theoretical optimization and real-time decision-making, further reinforcing the strategic utility of dynamic allocation models. (Francesco & Marco , 2012)

Ultimately, the convergence of classical optimization, behavioral finance, and modern computational techniques positions dynamic programming as a cornerstone of next-generation financial decision-making tools. By embedding these models into institutional frameworks, banks can achieve a more responsive, data-informed, and risk-conscious approach to loan allocation an essential capacity in an era of digital transformation and financial complexity.

Although the proposed model allows for an optimal allocation of resources based on purely economic and mathematical criteria, several discrepancies may arise when applied to real-world situations. In fact, the Moroccan banking system—like any financial environment—is subject to strict regulatory constraints imposed by Bank Al-Maghrib, particularly regarding prudential ratios, liquidity risk management, and credit concentration limits. Moreover, allocation decisions may be influenced by public policies, sectoral priorities (such as promoting investment in agriculture or real estate), or internal institutional considerations specific to each bank, including portfolio risk management or strategic preferences of decision-makers. These factors limit the direct applicability of the model and call for contextual adaptation when implementing it in practice.

7. CONCLUSION

This study aims to present a practical mathematical model that seeks to improve the efficiency of financial resource allocation within banking institutions by adopting dynamic programming as a quantitative tool capable of supporting optimal phased financial decision-making over multiple time periods. The model was developed to allocate the total loan portfolio balance, amounting to 549,920 million dirhams, across three successive time periods, achieving a balance between maximizing economic utility and minimizing potential risks.

The proposed model is based on a logarithmic utility function that represents the principle of diminishing marginal utility, whereby each increase in allocated resources yields progressively smaller additional utility, reflecting the realistic financial behavior of institutions. The model also takes into account the time value of money through a discount factor ($\beta = 0.95$), which expresses the preference for current consumption over the future, and an interest rate ($r = 0.05$) to calculate returns on unused balances in each time period.

The study relied on actual data issued by Bank Al-Maghrib for the third quarter of 2023, including details of loan balances distributed among three main types: equipment loans, mortgage loans, and consumer loans. The mathematical model was built using the power sum formula to calculate the relative weights for each time period according to the discount factor, and then determine the optimal allocation for each period based on these weights. The remaining balance after each stage was recalculated by adding the returns accrued from previous periods.

The model was practically implemented using the Python programming language, leveraging powerful analytical tools such as the NumPy, Matplotlib, and Pandas libraries, which enabled accurate and fast simulation of different allocation scenarios and provided quantitative results that can be relied upon for strategic decision-making.

The results showed that the largest portion of the balance is preferably allocated in the first period ($T1$), reflecting the priority of current consumption in banking institutions due to the preference for present time, followed by the second period ($T2$), and finally the third period ($T3$), which received the smallest portion of the balance. This distribution reflects the gradual decrease in allocations across the three periods due to the effects of the discount factor and interest rate.

The study also revealed noticeable gaps between the actual distribution of bank loans and the optimal distribution calculated according to the mathematical model, indicating potential improvements in the current financial policies followed by banking institutions. The model demonstrates how adopting precise quantitative methodologies can contribute to better utilization of available financial resources, reduce waste, and increase the efficiency of investment and financing decisions.

Despite the valuable insights provided, this study has several limitations. First, the model considers only three discrete time periods, which may not fully capture the complexity of financial planning over longer horizons. Additionally, the model assumes deterministic parameters for interest rates and discount factors, which may vary in real-world financial environments. The exclusion of risk factors and uncertainties, such as defaults or economic shocks, limits the model's ability to fully simulate realistic banking conditions. Furthermore, the model focuses exclusively on loan portfolio allocation, neglecting other financial assets or liabilities that might influence overall resource management.

Based on the study's results and observed limitations, several future directions and recommendations emerge. Extending the model to include a greater number of time periods ($T > 3$) would allow for more comprehensive long-term financial planning that better reflects evolving economic conditions. Incorporating stochastic elements through stochastic dynamic programming would enable the model to handle uncertainties and variability in parameters such as interest rates, repayment rates, and macroeconomic shocks. The integration of artificial intelligence and machine learning techniques could enhance the model's predictive power, adaptability, and automation, facilitating real-time decision support.

Moreover, testing the model across different financial products such as investments, government support programs, or non-loan assets would assess its broader applicability beyond banking loan portfolios. Embedding this dynamic programming model within the strategic financial planning frameworks of banking institutions could improve decision quality and operational efficiency. Equally important is the development of human capital: training financial analysts and decision-makers to understand, interpret, and apply quantitative model outputs is crucial for effective implementation.

The effective implementation of the optimal allocation model within the Moroccan banking context requires careful consideration of its feasibility. In practice, several local constraints must be taken into account, including the legal framework governing credit operations, regulatory requirements imposed by Bank Al-Maghrib, and specific prudential rules. Additionally, borrowing habits among Moroccan clients—often shaped by sociocultural factors—may limit the acceptance of a strictly optimized distribution. Furthermore, internal risk management policies specific to each banking institution may lead to the prioritization of certain loan categories for strategic reasons. These factors highlight the need to adapt the model to real-world conditions in order to ensure its successful integration into banking management practices.

A natural extension of the proposed model would involve integrating approaches from artificial intelligence, particularly reinforcement learning, to enable adaptive decision-making in a constantly evolving banking environment. This learning method, based on interaction with the environment, would allow the system to adjust resource allocations in response to observed outcomes (rewards), while gradually adapting to market changes, borrower behavior, and macroeconomic shifts.

Unlike traditional deterministic models, machine learning approaches incorporate uncertainty and randomness, making them especially suitable for complex financial systems. For example, Markov Decision Processes (MDPs) and deep neural networks can better model the dynamics of credit risk, loan repayment delays, and shifts in customer preferences.

In addition, supervised machine learning techniques—such as random forests, support vector machines, or neural networks—could be employed to predict default probabilities, expected profitability, or customer loyalty using historical data. These predictions could then be fed into the dynamic allocation model, enhancing its intelligence, responsiveness, and ability to anticipate market fluctuations.

In summary, integrating artificial intelligence into dynamic resource allocation in banking would enable a shift from static optimization to continuous, personalized, and learning-based optimization. Such an evolution represents a promising pathway to strengthen the resilience, profitability, and relevance of financial decision-making in the Moroccan banking sector.

REFERENCES

1. Al-Baz, M. A. (2005). Operations Research and Its Applications in Management. Dar Al-Safaa for Publishing and Distribution, Amman.
2. Al-Hilali, A. A. (2010). Quantitative Models in Administrative Decision Making. Dar Wael Publishing, Jordan.
3. Al-Jumaily, T. M. (2012). Decision Analysis Using Operations Research. University of Mosul – Iraq.
4. Al-Maghrib, B. (2023). Ventilation du crédit bancaire par objet économique (Déc. 2001 à sept. 2023) [jeu de données]. https://data.gov.ma/data/fr/dataset/ventilation-du-credit-bancaire-par-objet-economique-dec-2001-a-sept-2023
5. Arrow, K. J. (1971). Essays in the Theory of Risk-Bearing. *North-Holland Publishing Company*.
6. Bellman, R. (1957). Dynamic Programming. *Princeton University Press*.
7. Bellman, R. (1957). Dynamic Programming. *Princeton University Press*.
8. Bellman, R., & Dreyfus, S. (1962). Applied Dynamic Programming. Princeton University Press.
9. Bertsekas, D. P. (2005). Dynamic Programming and Optimal Control, Vol. I (3rd Edition). *Athena Scientific*.

10. Bertsekas, D. P. (2012). Dynamic Programming and Optimal Control, Volumes I & II. Athena Scientific.
11. Brealey, R., Myers, S., & Allen, F. (2020). Principles of Corporate Finance. McGraw-Hill Education.
12. Brealey, R., Myers, S., & Allen, F. (2020). Principles of Corporate Finance. McGraw-Hill Education.
13. Carlier, G. (2021). Programmation dynamique et mesures du risque. *École Polytechnique, Département de Mathématiques Appliquées*.
14. Coffee, J., & Jr, e. (2018). The Oxford Handbook of Banking. Oxford University Press.
15. Duffie, D., & Singleton, K. (2019). Credit Risk: Pricing, Measurement, and Management. Princeton University Press.
16. Dupont, L. (2017). Analyse du problème d'exécution optimale. *Mémoire de Master M17316, Université Paris-Dauphine, Département de Mathématiques Appliquées*.
17. Francesco , B., & Marco , C. (2012). Reinforcement Learning for Automatic Financial Trading: Introduction and Some Applications. *Working Paper No. 33/2012, Department of Economics, Ca' Foscari University of Venice*.
18. Frank, J., Fabozzi, Petter , N., Kolm, Dessislava , A., & Pa. (2010). Quantitative Investment Analysis. *John Wiley & Sons (2nd Edition), publié par le CFA Institute Investment Series*.
19. Glasserman, P. (2004). Monte Carlo Methods in Financial Engineering. *Springer*.
20. Goda, T. L. (2023). Dynamic Programming Approaches for Resource Allocation in Project Scheduling. *Journal of Operations Research and Applications*.
21. John, M., & Matthew , S. (2001). Learning to Trade via Direct Reinforcement. *IEEE Transactions on Neural Networks, 12(4), pages 875–889* .
22. Lasry, J.-M. (2016). Cours complet de programmation dynamique. *Lycée Janson de Sailly – Paris*.
23. Lieberman, H. (2021). Introduction to Operations Research. *McGraw-Hill Education*.
24. Ljungqvist, L., & Sargent, T. (2018). Recursive Macroeconomic Theory. MIT Press.
25. Merton, R. C. (1990). Continuous-Time Finance. *Blackwell Publishing*.
26. Nwozo, C., & Nkeki, C. (2012). On a dynamic optimization technique for resource allocation problems in a production company. *American Journal of Operations Research*.
27. Powell, W. B. (2011). Approximate Dynamic Programming: Solving the curses of dimensionali. Wiley.
28. Pratt, J. W. (1964). Risk Aversion in the Small and in the Large. *Econometrica, Vol. 32, No. 1-2, pp. 122–136*.
29. Puterman, M. L. (2005). Markov Decision Processes: Discrete Stochastic Dynamic Programming. *John Wiley & Sons*.
30. Puterman, M. L. (2014). Markov Decision Processes: Discrete Stochastic Dynamic Programming. Wiley.
31. Richard A, A., Brealey, Stewart , C., Myers, & Franklin All. (2019). Principles of Corporate Finance. *McGraw-Hill Education (13th Edition)*.
32. Santner, T. J. (1966). Optimal allocation of observations in two-stage designs. *Annals of the Institute of Statistical Mathematics*.
33. Stokey, N., Lucas, R., & Prescott, E. (1989). Recursive Methods in Economic Dynamics. Harvard University Press.
34. Varian, H. R. (2014). Intermediate Microeconomics: A Modern Approach. W. W. Norton & Company.
35. Whittle, P. (1982). Optimization over Time: Dynamic Programming and Stochastic Control. John Wiley & Sons.